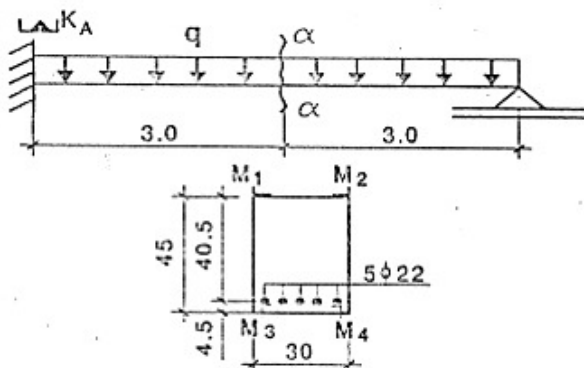




1. Na armiranobetonskom elementu, prema skici, merene su mernim trakama lokalne deformacije u preseku $\alpha-\alpha$ i promena nagiba deformacione linije u osloncu A. Rezultati merenja su dati tablično. Naći:

- naponsko stanje u preseku $\alpha-\alpha$
- opterećenje q .
- momenat elastičnog uklještenja u čvoru A.



st.	M_1	M_2	M_3	M_4	K_A
0	10153	12654	11136	11285	0+056
q	9780	12277	11766	11925	0+184

$$k_i = k, \quad \text{Č 240/360} \quad \text{MB 30}$$

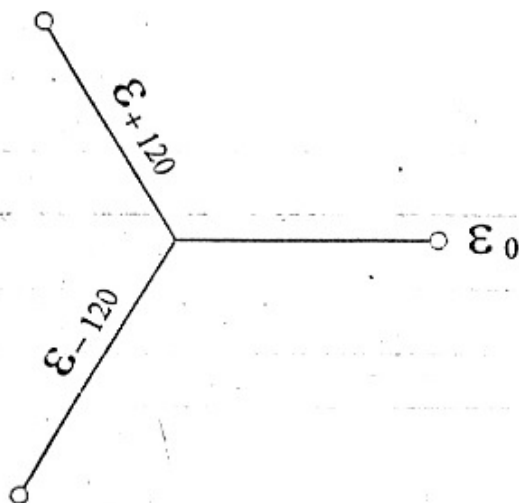
$$E_c = 2.1 \cdot 10^4 \text{ kN/cm}^2 \quad E_s = 0.3 \cdot 10^4 \text{ kN/cm}^2$$

$$A_s = 19.01 \text{ cm}^2 \quad (5 \phi 22)$$

30

2. Na čeličnom elementu je putem rozete, date na skici, mereno deformacijsko stanje u tački A. Računski i grafički odrediti:

- deformacijsko i naponsko stanje u tački A
- napon u pravcu koji je pod uglom od 90° u odnosu na 0-pravac
- nacrtati orijentaciju krsta glavnih napona u tački A, ako se 0-pravac poklapa sa horizontalom.



$$\epsilon_0 = 480 \cdot 10^{-6} \text{ mm/mm}$$

$$\epsilon_{+120} = -17 \cdot 10^{-6} \text{ mm/mm}$$

$$\epsilon_{-120} = -305 \cdot 10^{-6} \text{ mm/mm}$$

30

3. Na modelu zadanog prototipa, prema skici, izvršeno je merenje opštih i lokalnih deformacija pod probnim opterećenjem (dilatacije i uhib u sredini raspona od para sila $P_m = 10 \text{ kN}$), kao i frekvenciju slobodnog oscilovanja žica kablova za prethodno naprezanje ($f_{cr} = 151 \text{ Hz}$; $l_1 = 100 \text{ cm}$). Odrediti:

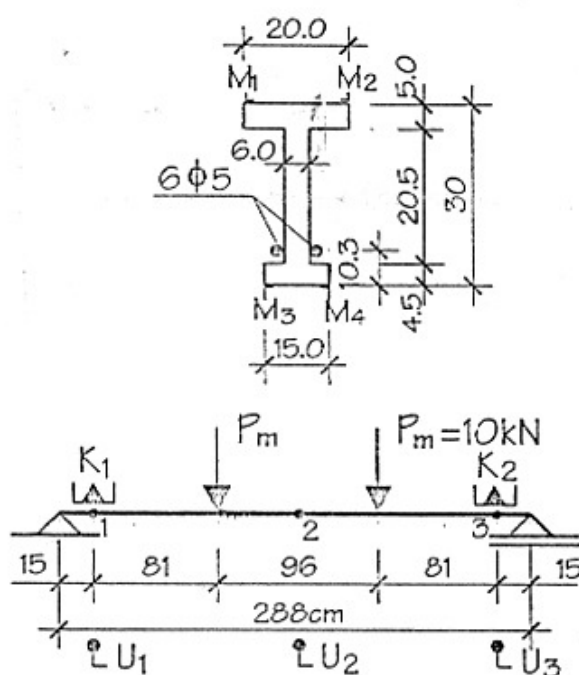
- Ukupan dijagram naprezanja (od probnog opterećenja i od prethodnog naprezanja)
- A zatim interpretirati dijagram naprezanja i maksimalni uhib na zadati prototip koristeći jednačine predviđanja za uhib i maksimalni normalni napon u sredini raspona. Takođe definisati sve ostale veličine prototipa koje nisu zadate.

Model i prototip su izrađeni od betona istih karakteristika.

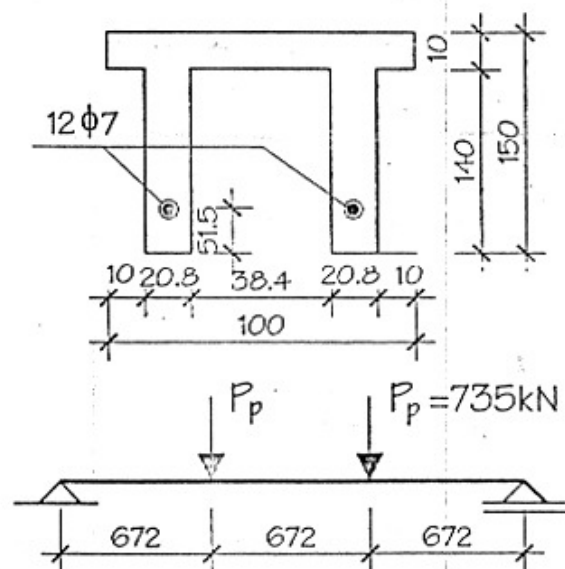
Stanje	U_1	U_2	U_3	K_1	K_2	M_1	M_2	M_3	M_4
0	1672	2021	1454	0+164	0+112	26437	26337	28809	26909
P	1698	2110	1484	0+207	0+248	26325	26225	28940	27040
0	1672	2022	1455	0+160	0+114	26435	26336	28810	26910

$$k_1 = k_2 \quad p_U = 0.01 \text{ mm}$$

Model:



Prototip:



$\lambda = 7$
 $\lambda_{p,cr} = 5$
 $\lambda_p = 77.5 = \frac{1}{\lambda} \cdot \lambda_{p,cr} \cdot \lambda^2$
 $\lambda_p = 1.5$

1. РЕЗУЛЬТАТЫ МЕТЕЛА:

	M1	M2	M3	M4	K _A
2-0	-373	-377	630	640	128
$\times 10^6$ E	-373	-377	630	640	135,62
E _R	-375		635		0,00066
σ_{R2}	-1,125		13,335		

$$\varepsilon = \frac{\mu}{\mu_0} \cdot \Delta c \cdot \rho \quad (\rho = 1 \cdot 10^{-6} \frac{\text{mm}}{\text{mm}})$$

$$\sigma_{1/2} = \varepsilon_{1/2} \cdot E_b$$

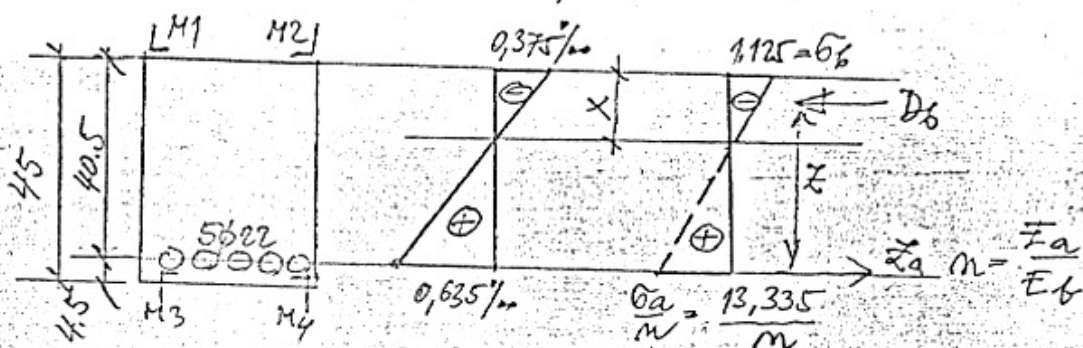
$$\sigma_{3/4} = \varepsilon_{3/4} \cdot E_c$$

$$E_s = 0,3 \cdot 10^4 \text{ kN/cm}^2$$

$$E_c = 2,1 \cdot 10^4 \text{ kN/cm}^2$$



а) НАПОННО-СТАВЬ У ПРЕСЕКУ:



$$x = \frac{E_b}{E_a + E_b} \cdot h = \frac{0,375}{0,635 + 0,375} \cdot 40,5 = 15,04 \text{ cm}$$

$$D_b = \frac{1}{2} \sigma_b \cdot x \cdot b = \frac{1}{2} \cdot 1,125 \cdot 15,04 \cdot 30 = 253,8 \text{ kN}$$

$$Z_a = \sigma_a \cdot A_a = 13,335 \cdot 19,01 = 253,5 \text{ kN}$$

б)

УСТОЯЧИВОСТЬ: $\sum N = 0$

$$x = h - \frac{x}{3} = 40,5 - \frac{1}{3} \cdot 15,04 = 35,5 \text{ cm}$$

МОМЕНТ У ПРЕСЕКУ $M = Z_a \cdot x = 253,5 \cdot 35,5 \cdot 10^{-2} = 89,99 = 90 \text{ kNm}$

ДИНАМИЧЕСКИЙ МОМЕНТ: $\frac{2l^2}{8} = \frac{M_A}{2} + M$

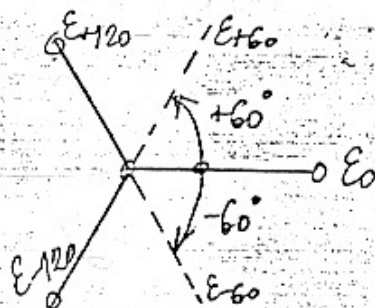
$$M_A = \frac{2l^2}{8} - \frac{3EI}{l} \cdot \frac{1}{10} \Delta t = \frac{2l^2}{8} - \frac{3 \cdot 0,3 \cdot 10^8}{6,0} \cdot \frac{1}{12} \cdot 0,3 \cdot 0,45^3 \cdot 0,00066$$

$$M_A = \frac{2l^2}{8} - 22,5$$

$$\frac{2l^2}{8} = \frac{2l^2}{16} - \frac{22,5}{2} + 90 \Rightarrow \frac{2l^2}{16} = 78,75 \Rightarrow 2 \cdot \frac{78,75 \cdot 16}{36} = 35 \frac{\text{kN}}{\text{m}^2}$$

в) $M_A = \frac{1}{8} \cdot 35 \cdot 36,0 - 22,5 = 135 \text{ kNm}$

20



$$E_0 = 480 \cdot 10^{-6} \frac{\text{В}}{\text{м}}$$

$$E_{+120} = -17 \cdot 10^{-6} \frac{\text{В}}{\text{м}} = E_{-60}$$

$$E_{-120} = -305 \cdot 10^{-6} \frac{\text{В}}{\text{м}} = E_{+60}$$

I РАСЧЕТНЫЙ ПОСТУПАК:

$$a) E_{1,2} = \frac{E_0 + E_{60} + E_{-60}}{3} \pm \frac{1}{3} \sqrt{(2E_0 - E_{60} - E_{-60})^2 + 3(E_{60} - E_{-60})^2}$$

$$10^6 E_{1,2} = \frac{480 - 17 - 305}{3} \pm \frac{1}{3} \sqrt{(2 \cdot 480 + 17 + 305)^2 + 3(-17 + 305)^2} = 52,6 \pm \frac{1}{3} \sqrt{2922^2}$$

$$10^6 E_{1,2} = 52,6 \pm 458,543 \Rightarrow E_1 = 511,210 \cdot 10^{-6} \frac{\text{В}}{\text{м}}$$

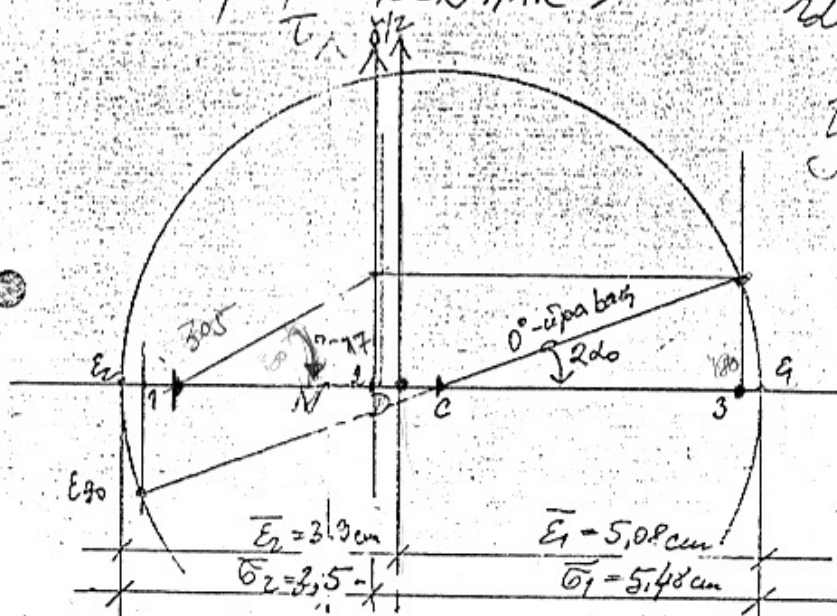
$$E_2 = -405,876 \cdot 10^{-6} \frac{\text{В}}{\text{м}}$$

$$\cos 2\alpha^* = \left| \frac{E_{60} - E_{-60}}{2E_0 - E_{60} - E_{-60}} \right| \cdot \sqrt{3} = 0,3891 \Rightarrow \alpha^* = 10,63^\circ = \alpha_0 \begin{cases} 57^\circ \\ 127^\circ \end{cases}$$

$$\sigma_1 = \frac{E}{1-\nu^2} (E_1 + \nu E_2) = \frac{21 \cdot 10^4}{1-0,3^2} (511,210 - 0,3 \cdot 405,876) \cdot 10^{-6} = 8,987 \frac{\text{кН}}{\text{см}^2}$$

$$\sigma_2 = \frac{E}{1-\nu^2} (E_2 + \nu E_1) = \frac{21 \cdot 10^4}{1-0,3^2} (-405,876 + 0,3 \cdot 511,210) \cdot 10^{-6} = -5,827 \frac{\text{кН}}{\text{см}^2}$$

II ГРАФИЧЕСКИЙ ПОСТУПАК:



$$r_d = 1 \text{ см} \triangleq 100 \cdot 10^{-6} \frac{\text{В}}{\text{м}}$$

$$r_n = \frac{21 \cdot 10^4}{1-0,3} \cdot 100 \cdot 10^{-6} = 1,615 \frac{\text{В}}{\text{см}^2}$$

$$CD = \frac{52,6}{100} = 0,53 \text{ см}$$

$$CO = \frac{1,3}{0,7} \cdot 0,53 = 0,958 \frac{\text{В}}{\text{см}^2}$$

$$E_1 = 5,08 \cdot 100 \cdot 10^{-6} = 508 \cdot 10^{-6} \frac{\text{В}}{\text{м}}$$

$$E_2 = -3,3 \cdot 100 \cdot 10^{-6} = -330 \cdot 10^{-6} \frac{\text{В}}{\text{м}}$$

$$\sigma_1 = 5,48 \cdot 1,615 = 8,85 \frac{\text{кН}}{\text{см}^2}$$

$$\sigma_2 = -3,50 \cdot 1,615 = -5,65 \frac{\text{кН}}{\text{см}^2}$$

$$2\alpha_0 = 20,5^\circ \Rightarrow \alpha_0 = 10,25^\circ$$

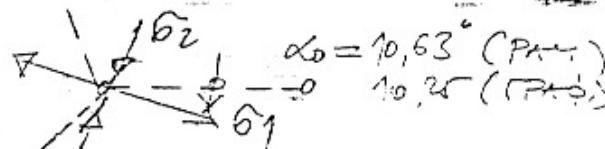
3) РАСЧЕТНЫЙ:

$$E_{90} = \frac{E_1 + E_2}{2} - E_1 E_2 \cdot \cos 2\alpha_0 = 52,667 \cdot 10^{-6} - 458,543 \cdot \cos(2 \cdot 10,63^\circ) \cdot 10^{-6} = -374 \cdot 10^{-6} \frac{\text{В}}{\text{м}}$$

$$\sigma_{90} = \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 - \sigma_2}{2} \cdot \cos 2\alpha_0 = 1,58 - 7,407 \cdot \cos(2 \cdot 10,63^\circ) = -5,323 \frac{\text{кН}}{\text{см}^2}$$

ГРАФИЧЕСКИЙ: $E_{90} = -3,5 \cdot 100 \cdot 10^{-6} = -350 \cdot 10^{-6} \frac{\text{В}}{\text{м}}$ $\sigma_{90} = -3,5 \cdot 1,615 = -5,65 \frac{\text{кН}}{\text{см}^2}$

1)



3. СРАВНАВАМЕ МЕРЕНИХ ПОДАТАКА :

СТ	u ₁	u ₂	u ₃	K ₁	K ₂	M ₁	M ₂	M ₃	M ₄
ΔC ₁	26	89	30	137	136	-112	-112	131	131
ΔC ₂	26	88	29	133	134	-110	-111	130	130
ΔC _{cp}	26	88,5	29,5	135	135	-111	-111,5	130,5	130,5
УТ.	0,26	0,885	0,295	143,1	143,1	-111	-111,5	130,5	130,5

$$\begin{aligned}
 u &= p_n \cdot \Delta \bar{C}_{sr} ; p_n = 0,01 \text{ мм} & [u] &= \text{мм} \\
 \alpha &= p_k \cdot \Delta \bar{C}_{sr} ; p_k = 1,06'' & [\alpha] &= '' \\
 \varepsilon &= \frac{k_i}{k_t} \cdot p \cdot \Delta \bar{C}_{sr} ; p = 1 \cdot 10^{-6} \frac{\text{мм}}{\text{мм}} ; \frac{k_i}{k_t} = 1 ; [\varepsilon] = 10^{-6} \frac{\text{мм}}{\text{мм}} \\
 & & G &= \pm 6 \cdot C_6 \\
 \delta \alpha_1 &= 6,938 \cdot 10^{-4} = \delta \alpha_2
 \end{aligned}$$



Бреј до стабилна
иога брзак и то
брат

$$\begin{aligned}
 u_A &= u - h_A \\
 h_A &= 15,0 \cdot \delta \alpha_1
 \end{aligned}$$

$$h_A = 15 \cdot 6,938 \cdot 10^{-4} = 0,01 \text{ см} = h_B$$

$$u_A = 0,026 - 0,01 = 0,016 \text{ см}$$

$$u_B = 0,0295 - 0,01 = 0,0195 \text{ см}$$

$$u_{2 \text{ сред}} = u_2 - \frac{u_A + u_B}{2} = 0,0885 - \frac{0,016 + 0,0195}{2} = 0,071 \text{ см}$$

$$u_{2 \text{ пр}} = \frac{23}{648} \cdot \frac{p \ell^3}{J \cdot E}$$

$$\pm \frac{u}{6} = \frac{23}{648} \cdot \frac{p_n \cdot h_n^3}{J_m \cdot u_{2 \text{ сред}}}$$

- ГЕОМЕТРИЈСКЕ КАРАКТЕРИСТИКЕ МОДЕЛА :

$$A_{\text{ком}} = 20 \cdot 5 + 20,5 \cdot 6 + 15 \cdot 4,5 = 100 + 123 + 67,5 = 290,5 \text{ см}^2$$

$$\begin{aligned}
 J_{\text{ком}} &= \frac{1}{12} (20 \cdot 5^3 + 6 \cdot 20,5^3 + 15 \cdot 4,5^3) + 100 \cdot (13,8 - 2,5)^2 + 123 \cdot (15,25 - 5)^2 \\
 &+ 67,5 \cdot (24,75 - 13,8)^2 = 11629,8 + 12769 + 258,61 + 13135,67 = \\
 &= 30793,08 \text{ см}^4
 \end{aligned}$$

$$Y_{Tmg} = \frac{100 \cdot 21,5 + 123 \cdot 15,25 + 67,5 \cdot 27,75}{290,5} = 13,8 \text{ cm}$$

$$Y_{Tmd} = 30 - 13,8 = 16,2 \text{ cm}$$

$$W_{g_m} = \frac{30793,08}{13,8} = 2231,4 \text{ cm}^3$$

$$W_{d_m} = \frac{30793,08}{16,2} = 1900,81 \text{ cm}^3$$

$$F_{bm} = \frac{23}{648} \cdot \frac{10 \cdot 288^3}{30793,08 \cdot 0,071} = 3878 \text{ kN/cm}^2 \quad 5$$

a) - Находиш ли елине σ_{mi} ;

$$\sigma_1 = \varepsilon \cdot F_1 = -111 \cdot 10^{-6} \cdot 3878 = -0,43 \text{ kN/cm}^2$$

$$\sigma_2 = -111,5 \cdot 10^{-6} \cdot 3878 = -0,432 \text{ kN/cm}^2$$

$$\sigma_3 = 130,5 \cdot 10^{-6} \cdot 3878 = 0,506 \text{ kN/cm}^2 - \sigma_4$$

- Находиш ли елине брзоходног напрезања:

$$\sigma_z = c \cdot \rho^2 \cdot f_{ex}^2 = 3,2 \cdot 10^{-7} \cdot 100^2 \cdot 151^2 = 72,96 \text{ kN/cm}^2$$

$$N_k = A_k \cdot \sigma_z = 2 \cdot 6 \cdot \frac{0,5^2 \pi}{4} \cdot 72,96 = 171,91 \text{ kN}$$

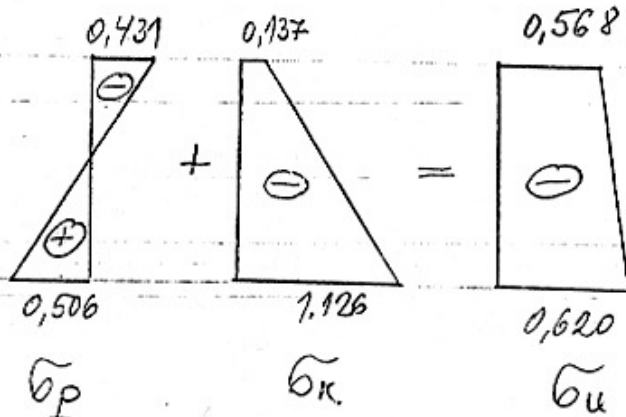
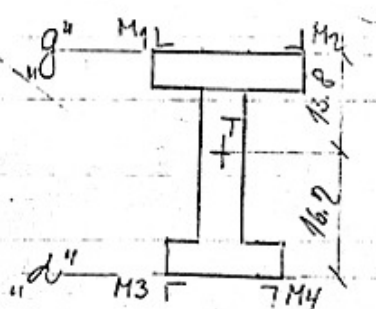
$$e_k = 16,2 - 10,3 = 5,9 \text{ cm}$$

$$\sigma_{k,ol} = -\frac{N_k}{A_b} \pm \frac{N_k e_k}{W_{b,ol}}$$

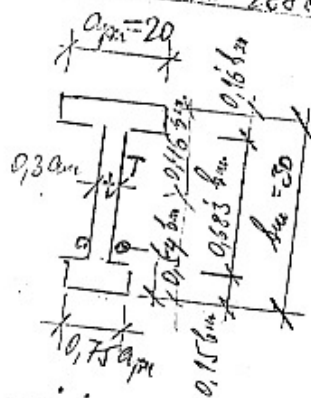
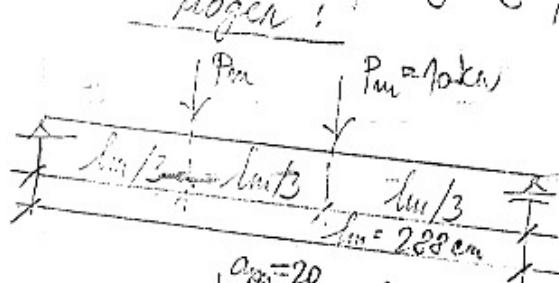
$$\sigma_{k,g} = -\frac{171,91}{290,5} + \frac{171,91 \cdot 5,9}{2231,4} = -0,592 + 0,455 = -0,137 \text{ kN/cm}^2$$

$$\sigma_{k,ol} = -\frac{171,91}{290,5} - \frac{171,91 \cdot 5,9}{1900,81} = -0,592 - 0,534 = -1,126 \text{ kN/cm}^2 \quad 5$$

- УКУПАН ЛИЈАТРАМ НАПРЕЗАЊА:



Оптимизација



$$I_{m0} = 0.15 \cdot 0.15^3 + 0.3 \cdot 0.15 \cdot 0.683^2 + 0.15 \cdot 0.15 \cdot 0.683^2 = 0.4842 \text{ am}^4$$

$$J_m = 0.0570 \cdot \text{am}^4$$

$$W_m^g = \frac{0.0570 \text{ am}^4}{0.46 \text{ am}} = 0.124 \text{ am}^3$$

$$W_m^d = \frac{0.057 \text{ am}^4}{0.54 \text{ am}} = 0.106 \text{ am}^3$$

факторы:

$$\lambda_A = \frac{I_p}{I_m} = \frac{0.4549 a_p \cdot b_p}{0.4842 \text{ am}^4} = 0.9395 \frac{\text{m}^2}{\text{cm}^4}$$

$$\lambda_J = \frac{J_p}{J_m} = 0.7444 \frac{\text{m}^4}{\text{cm}^4}$$

$$\lambda_W = \frac{W_{pol}}{W_{mol}} = 0.7412 \frac{\text{m}^3}{\text{cm}^3}$$

$$\mu = \frac{23}{648} \cdot \frac{Pl^3}{EJ}$$

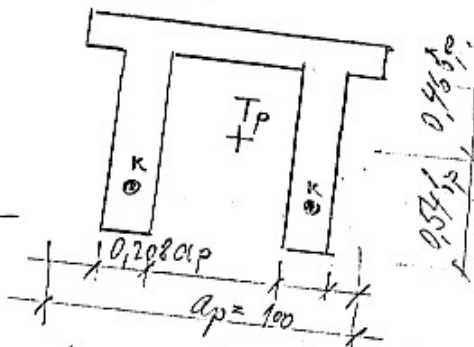
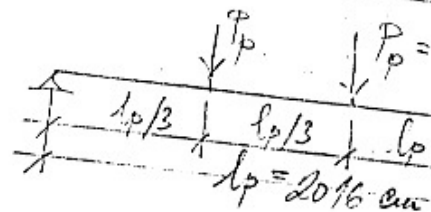
$$\frac{\mu_p}{l_p} = \frac{23}{648} \cdot \frac{P_p}{E_p \cdot l_p^2} \cdot \frac{l_p^4}{E_p}$$

$$\frac{\mu_m}{l_m} = \frac{23}{648} \cdot \frac{P_m}{E_m \cdot l_m^2} \cdot \frac{l_m^4}{J_m}$$

$$\delta_{ol} = -\frac{N_k}{A} - \frac{N_k \cdot e_k}{W_{ol}} + \frac{Pl}{3 \cdot W_{ol}}$$

$$\frac{\delta_p}{l_p^2} = \frac{1}{3} \cdot \frac{l_p^3}{W_{pol}} - \frac{N_{k,p}}{P_p} \cdot \frac{l_p^2}{A_p} \left(1 + \frac{A_p \cdot e_{k,p}}{W_{pol}}\right)$$

$$\frac{\delta_{ol,m}}{P_m/l_m^2} = \frac{1}{3} \frac{l_m^3}{W_{mol}} - \frac{N_{k,m}}{P_m} \cdot \frac{l_m^2}{A_m} \left(1 + \frac{A_m \cdot e_{k,m}}{W_{mol}}\right)$$



$$I_p = 0.1 \cdot 0.1^3 + 0.298 \cdot 0.1 \cdot 0.463^2 = 0.4549 \text{ ap}^4$$

$$J_p = 0.0005 + 0.0012 \cdot 2 \cdot 80 = 0.0029$$

$$e_{tol} = 150 - 69 = 81 \text{ cm} = 0.81 \text{ m}$$

$$J_p = 0.04243 \text{ ap}^4$$

$$W_{pg} = 0.0922 \text{ ap}^3$$

$$W_{pol} = 0.07857 \text{ ap}^3$$

$$\frac{e_{k,p}}{a_m} = \frac{b_p}{l_m} = 5$$

- істотні розбіжності:

$$\frac{\frac{U_p}{l_p}}{\frac{U_m}{l_m}} = \frac{\frac{23}{648} \cdot \frac{P_p}{E_p l_p^2} \cdot \frac{l_p^4}{J_p}}{\frac{23}{648} \cdot \frac{P_m}{E_m l_m^2} \cdot \frac{l_m^4}{J_m}} \quad (1) \quad \frac{\frac{\sigma_{dp}}{P_p / l_p^2}}{\frac{\sigma_{dm}}{P_m / l_m^2}} = \frac{\frac{1}{3} \frac{l_p^3}{W_{dp}} - \frac{N_{dp}}{P_p} \cdot \frac{l_p^2}{J_p} \left(1 + \frac{C_d}{C_w} \frac{A_{m \text{ вив}}}{W_{dp}}\right)}{\frac{1}{3} \frac{l_m^3}{W_{dm}} - \frac{N_{dm}}{P_m} \cdot \frac{l_m^2}{J_m} \left(1 + \frac{C_d}{C_w} \frac{A_{m \text{ вив}}}{W_{dm}}\right)} \quad (2)$$

$$(1) \Rightarrow \frac{U_p}{U_m} = r_u = \frac{P_p}{P_m} \cdot \frac{E_m}{E_p} \cdot \left(\frac{l_m}{l_p}\right)^2 \cdot \frac{J_m}{J_p} \cdot \left(\frac{l_p}{l_m}\right)^4 =$$

$$= r_p \cdot \frac{1}{r_E} \cdot \frac{1}{r_l^2} \cdot \frac{1}{r_J} \cdot r_l^4 = \frac{r_p \cdot r_l^2}{r_E \cdot r_J}$$

$$r_E = \frac{E_p}{E_m} = 1$$

$$r_p = \frac{P_p}{P_m} = r_E \cdot r_l^2 \cdot r_J^2 = \frac{735}{10} = 73.5 \Rightarrow$$

$$\Rightarrow r_E = 1.5$$

$$r_m = \frac{r_E \cdot r_l^4}{C_J \cdot r_{p \text{ пор}}} = \frac{1.5 \cdot 7^4}{0.7444 \cdot 5^4} = 7.741 \quad (\text{розміра за гнуче})$$

$$(2) \Rightarrow \frac{\frac{\sigma_{dp}}{\sigma_{dm}} \cdot \left(\frac{l_p}{l_m}\right)^2 \cdot \frac{P_m}{P_p}}{\frac{1}{3} \frac{l_m^3}{W_{dm}} - \frac{N_{dm}}{P_m} \cdot \frac{l_m^2}{J_m} \left(1 + \frac{C_d}{C_w} \frac{A_{m \text{ вив}}}{W_{dm}}\right)} = \frac{\frac{r_l^3}{C_w \cdot r_{p \text{ пор}}} \cdot \left(\frac{1}{3} \frac{l_m^3}{W_{dm}}\right) - \frac{N_{dm}}{P_m} \cdot \frac{l_m^2}{J_m} \left(1 + \frac{C_d}{C_w} \frac{A_{m \text{ вив}}}{W_{dm}}\right) \cdot \frac{r_l^2}{C_d \cdot r_{p \text{ пор}}}}{\frac{1}{3} \frac{l_m^3}{W_{dm}} - \frac{N_{dm}}{P_m} \cdot \frac{l_m^2}{J_m} \left(1 + \frac{C_d}{C_w} \frac{A_{m \text{ вив}}}{W_{dm}}\right)}$$

$$r_l \cdot r_l^2 \cdot \frac{1}{r_{p \text{ пор}}} = \frac{\frac{r_l^3}{C_w \cdot r_{p \text{ пор}}} \cdot 4973.283 - 17.191 \cdot 285.502 \left(1 + \frac{C_d}{C_w} \cdot 0.6981\right) \cdot \frac{r_l^2}{C_d \cdot r_{p \text{ пор}}}}{-5144.1874}$$

$$r_l = \left(0.954 \frac{C_w}{C_d} + 0.8572 - 0.8113 \frac{r_u}{r_{p \text{ пор}}} \right) \cdot \frac{r_E \cdot r_l^2}{C_w \cdot r_{p \text{ пор}}}$$

$$r_l = \left(0.954 \cdot \frac{0.7412}{0.9395} + 0.8572 - 0.8113 \cdot \frac{7}{5} \right) \cdot \frac{1.5 \cdot 7^2}{0.7412 \cdot 5^2} = 1.88$$

ГНУБ ПРОТОТИПА:

$$U_p = r_m \cdot U_m = 7.741 \cdot 0.071 = 0.55 \text{ см}$$

НАПОРИ ПРОТОТИПА:

$$\sigma_{dp} = -1.88 \cdot 0.568 = -1.068 \text{ кн/см}^2$$

$$\sigma_{dp} = -1.88 \cdot 0.620 = -1.166 \text{ кн/см}^2$$

СИЛА ПРІТХОДНОГО НАПРЕЗАННЯ ПРОТОТИПА:

$$N_{dp} = 73.5 \cdot 171.91 = 12635.4 \text{ кн}$$

$v < 1.2 \text{ м/с}^2$ - дозв.
за 1.1.30